Applications of elementary probability theory in poker
A short introduction

Jason A. Osborne, Dept. of Statistics, NCSU
osborne@stat.ncsu.edu
www.stat.ncsu.edu/~jaosborn/research/poker.pdf

The different possible categories of hands are enumerated below, in decreasing order of probability under "stud" poker, where no draws are made after the initial 5 cards are dealt are given in the table below.

The number of ways to order 5 distinct items is $5! = 1*2*3*4*5$. For example, the $3! = 6$ ways to order the three items labeled A, B, C are

$$ABC, ACB, BAC, BCA, CBA, CAB.$$

The number of ways to choose 5 items from 52 distinct items, without replacement and without regard to the order in which they are chosen, is given by

$$\binom{52}{5} = \frac{52!}{5!(52-5)!} = 2,598,960.$$

Hand	Number of combinations	Probability	Empirical frequency
One Pair	$\binom{4}{2}\binom{13}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{12}{3}$	0.422569	0.42057
Two Pair	$\binom{4}{2}\binom{4}{2}\binom{13}{2}\binom{44}{1}$	0.0475	0.04669
Three-of-a-kind	$\binom{4}{3}\binom{13}{1}\binom{4}{1}\binom{4}{1}\binom{12}{2}$	0.0211	0.02096
Straight	$\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{10}{1}$	0.00394	0.00356
Non-flush straight	$\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{10}{1} - 40$	0.00392	0.00355
Flush	$\binom{13}{5}\binom{4}{1}$	0.00198	0.00196
Non-straight flush	$\binom{13}{5}\binom{4}{1} - 40$	0.00197	0.00195
Full house	$\binom{4}{3}\binom{13}{1}\binom{4}{2}\binom{12}{1}$	0.00144	0.00172
Four-of-a-kind	$\binom{?}{?}\binom{?}{?}\binom{?}{?}\binom{?}{?}$	0.000024	0.00029
Straight Flush	$\binom{5}{5}\binom{10}{1}\binom{4}{1} = 40$	0.0000154	0.00001
Royal Flush	$\binom{5}{5}\binom{4}{1}$	0.00000153	NA

Common denominator is $\binom{52}{5} = 2,598,960$.
Empirical frequencies are from $n = 100,000$ simulated hands.